IMPLEMENTATION NON-IDEALITY INFLUENCE ON THE NONLINEAR DYNAMICS OF CHAOTIC GENERATORS

BY

CARMEN GRIGORAȘ1,2 and VICTOR GRIGORAȘ3*

1“Gr.T. Popa” University of Medicine and Pharmacy of Iași
2Institute of Computer Science, Romanian Academy, Iaşi
3“Gheorghe Asachi” Technical University of Iaşi

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Abstract. The present contribution analyzes the influence of a type of algebraic nonlinearity in the circuit implementation of the Rossler system, on the nonlinear dynamics of this chaotic generator. The analogue multiplier, imposed by the nonlinearity in the Rossler state equations, is implemented using a bipolar junction transistor Gilbert cell. Its nonlinear input-output characteristic is reviewed, the modification of the system state equations due to this nonlinearity is presented and in depth simulations, highlighting its influence on the nonlinear dynamics of the Rossler system, are performed.

Key words: chaos generators, analog multipliers, Gilbert cell, nonlinear dynamics.

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1. Introduction

Noise generation using chaotic systems is of great interest due to its applications in test and measurement for communication equipment, communication security and spread spectrum clock generation. Recent research

*Corresponding author; e-mail: grigoras@etti.tuiasi.ro
results are reported using analog implementations (Stojanovsky & Kocarev, 2001; Udaltsov et al., 2002, Yalcin et. al., 2004, Grigoraş & Grigoraş, 2013),
digital systems (Leon et al., 2001; Yang et al., 2004; Grigoraş & Grigoraş, 2011; Grigoraş & Grigoraş, 2012) or a combination thereof (Andreatos &
Volos, 2014). Due to the fact that analog integrated circuit (IC) implementations
typically use more silicon area on a chip, finding the simplest nonlinear systems
for analog noise generators may find useful applications.

Simplifying the nonlinearities in the circuit may be achieved by using
multiplication in the system state equations. Classical chaotic systems
include single multiplier systems: Rossler (1976; 1979), Genesio (Genesio &
Tesi, 1992) and Coullet (Coullet et. al., 1979), the Lorenz (1963) and Chen
(Chen & Ueta, 1999; Ueta & Chen, 2000) systems, that use two multipliers
and a recently proposed three multiplier chaotic generator (Grigoraş &
Grigoraş, 2014).

In order to approach the IC implementation aspects for analogue noise
generators, we present a parametric analysis of a single multiplier type system,
to highlight the influence of the algebraic nonlinear non-idealities in the circuit
implementation of the system equations. The example of the Rossler system is
analyzed due to its simplicity. The presence of a single multiplication in the
state equations of the Rossler system leads to the necessity of a single analogue
multiplier in the circuit implementation helping in focusing the parametric
analysis to this building block. The analogue multiplier is implemented using a
bipolar junction transistor Gilbert cell. Its nonlinear input-output characteristic
is reviewed, the modification of the system state equations due to this
nonlinearity is presented and in depth analysis of the its influence on the
nonlinear dynamics of the Rossler system is performed. The parametric analysis
is performed using the bifurcation diagram method, at the variation of a scaling
factor, introduced in order to control the dynamic range of the signals at the
multiplier inputs. Finally, we verify by simulation the influence of the non-ideal
multiplier on the statistical characteristics of the signals generated by the
Rossler system, based on estimates of the probability distribution function and
power spectral density.

2. The System Model

The Rossler nonlinear system was first introduced in 1976 (Rossler,
1976), as an alternative to the Lorenz chaotic attractor. The normalized system
eqs. (1) show an analogue, third order, single multiplier type nonlinear system.

\[
\begin{align*}
x' &= -y - z \\
y' &= x + ay \\
z' &= b - cz + xz
\end{align*}
\]
Depending on the particular values of the equations coefficients, $a$, $b$ and $c$, the Rossler nonlinear system exhibits a variety of dynamical behaviors, including constant, periodic and chaotic dynamics.

The simplicity of the Rossler differential equations makes its electronic IC implementation attractive. For such an attempt, we need to perform further analysis of the system, with more accurate models of the analogue building blocks. In the following, we analyze the nonlinear dynamics of the Rossler nonlinear system implemented with a bipolar junction transistor Gilbert cell analog multiplier.

The basic Gilbert cell is composed of a three differential stage structure. In the case of a bipolar junction transistor circuit, the algebraic nonlinear function of the analog multiplier is given in eq. (2), where $V_T$ denotes the thermal voltage, $I_S$ the bias current source and $\tanh()$ the hyperbolic tangent function:

$$f(x, y) = I_S \cdot \tanh(x / V_T) \cdot \tanh(y / V_T)$$  (2)

The following results focus on estimating the ‘degree of nonlinearity’ at the multiplier input by down/up - scaling the input and output signals of the basic Gilbert cell, using the factor $K$:

$$f(x, y) = \tanh(K \cdot x) \cdot \tanh(K \cdot y) / K^2$$  (3)

The $K$ parameter in the eq. (3), includes the technological constant, $V_T$, and the circuit bias, $I_S$. Finally, the state equations of the Rossler nonlinear system, including the algebraic nonlinear non-idealities of the bipolar junction transistor Gilbert cell analog multiplier are obtained combining eqs. (1) and (3).

$$\begin{align*}
x' &= -y - z \\
y' &= x + a \cdot y \\
z' &= b - c \cdot z + \tanh(K \cdot x) \cdot \tanh(K \cdot z) / K^2
\end{align*}$$  (4)

Eqs. (4) are normalized, so de-normalization, with a dimensional factor, denoted $\omega_0$ in the following, may be performed before circuit implementation, based on the time scaling equation:

$$t_n = \omega_0 \cdot t \Rightarrow d t_n = \omega_0 \cdot d t$$  (5)

This leads to the de-normalized state equations:

$$\begin{align*}
x' &= -\omega_0 \cdot (y + z) \\
y' &= \omega_0 \cdot (x + a \cdot y) \\
z' &= \omega_0 \cdot \left( b - c \cdot z + \tanh(K \cdot x) \cdot \tanh(K \cdot z) / K^2 \right)
\end{align*}$$  (6)
The de-normalized system (6) is also useful in scaling the frequency bandwidth of the generated noise.

3. Simulation Results

We performed our parametric analysis based on the normalized eqs. (4) of the Rossler system. Our starting point was identifying the normalized coefficients value ranges, for different types of dynamical behaviors. To achieve this goal, bifurcation diagrams, such as those given in Fig. 1, were depicted around the nominal values used in the initial paper (Rossler, 1976):

\[ a = 0.2, \quad b = 0.2, \quad c = 5.7 \]

(7)

The main goal of our analysis was to identify the value ranges of the scaling factor \( K \) that lead to different nonlinear dynamical behaviors, for coefficients \( a, b \) and \( c \) having different values. As a result a ‘degree of nonlinearity’ image is expected. In this sense for small \( K \) values, the input signals for the multipliers have smaller amplitudes, leading to a better linear approximation of the hyperbolic tangent function and thus a dynamical behavior closer to that of the ideal Rossler system (1). Larger \( K \) values lead to a larger dynamic range for the hyperbolic tangent argument and thus a more important influence of the multiplier non-ideality, that may change the implemented system (4) dynamics referenced to the ideal one.

Fig. 1 − Bifurcation diagrams for the Rossler system at the variation of the coefficients \( a, b \) and \( c \).
As an example, in Fig. 2, the starting point of the parametric analysis is the standard chaotic set of coefficients (7). Expectedly, for small $K$ values, the chaotic behavior is preserved, but increasing the scaling factor over the 0.02 value shows periodic dynamics, with multiplicities evolving from 12 to 6 and finally 3. Confirming the rule ‘period three leads to chaos’, further increasing the scaling factor over the 0.03 bursts into a new chaotic region, that becomes periodic again for $K > 0.05$. This time the multiplicity series is the standard 8, 4, 2, 1. For values over the threshold 0.09, the periodic oscillations exponentially increase to infinity, showing that the used model is no longer valid.

![Bifurcation Diagram for $a = 0.2, b = 0.2, c = 5.7$.](image1)

![Bifurcation Diagram for $a = 0.3, b = 0.3, c = 5.5$.](image2)
The case presented in Fig. 3 uses a set of system coefficients, \( a = 0.3, \ b = 0.3, \ c = 5.5 \), for which the ideal Rossler system (1) exhibits periodic behavior. For small \( K \) values, the non-ideal system (4) preserves the periodic dynamics, with period multiplicity two. Slightly increasing the value of \( K \) leads to chaos for a large range of values, from 0.007 to 0.072, with short periodicity intervals. Scaling factor values over 0.072 lead to periodic behavior with period multiplicities 8, 4, 2, 1.

A final example is depicted in Fig. 4, taking the system coefficients \( a = 0.19, \ b = 0.3, \ c = 5.0 \) for which the Rossler system (1) yields periodic dynamics with high period multiplicity of 16. The period multiplicity rapidly decreases to 6, bursting into chaos for \( K \) values ranging from 0.01 to 0.025. After that, the period multiplicity regains the value 16 then slowly decreasing in a 8, 4, 2, 1 series. Like in all previous examples, values over 0.09 lead to exponential increase of the periodic oscillations amplitude, suggesting the need of a more refined model that takes into account supply voltage limitation.

The next part of our simulations aims at checking the influence of the non-ideal multiplier on the performance of the Rossler system aimed at a noise generation application. Estimates of the first and second order statistics for the system state variables were calculated, at the variation of the scaling factor, \( K \), inside the value range ensuring chaotic dynamics.

The probability density was estimated by use of the histogram method. The dynamic range of each state variable of the Rossler system was uniformly partitioned into \( N = 100,..., 250 \) subintervals and the frequency of the trajectory visitation of each subinterval is taken as an estimate of the probability density. The graphs depicted in Fig. 5 show such an example, for the state variable \( x_2 \).
and a number of 120 subintervals. In this example, we may notice a stronger influence of the scaling factor variation on the probability density function estimate, especially at larger values, in the case of system parameters \( a = 0.3, \ b = 0.3 \) and \( c = 5.5 \) than for the standard coefficient set (7).

We estimated the power spectral density of the state variables using the periodogram method. In the logarithmic representations exemplified in Fig. 6, we may notice that variation of the scaling factor leads to less important power spectral density modifications, especially from the qualitative point of view. For noise generation applications, the power spectral density is relatively flat a for very low frequency band, 0,…, 0.5 Hz. If coloured noise is acceptable, the frequency bandwidth up to 5 Hz may be used, with a roll-off speed of 20 dB/decade.

![Figure 5](image.png)

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Fig. 5 − Comparison of histograms at the variation of the scaling factor: continuous line for ideal system, dash-dot line for \( K = 0.015 \), dashed line for \( K = 0.031 \) and dotted line for \( K = 0.045 \); up \( a = 0.3, \ b = 0.3 \) and \( c = 5.5 \), down \( a = 0.2, \ b = 0.2 \) and \( c = 5.7 \); for both graphs.
Fig. 6 − Comparison of the power spectral density of the third state variable, for \(a = 0.2\), \(b = 0.2\) and \(c = 5.7\), at the variation of the scaling factor: 
- A) ideal system, B) \(K = 0.015\), C) \(K = 0.031\) and D) \(K = 0.045\).

4. Conclusions

Analyzing the nonlinear dynamics of the Rossler system, with account of the algebraic non-idealities in the analog multiplier implementation, we showed that, for conveniently chosen system parameters and scaling factors, we can preserve the dynamic behavior of the ideal system. For system parameters ensuring chaotic dynamics, it is noticeable that increasing the scaling factor the implementable system becomes periodic. Moreover, we noticed that the nonlinear dynamics of the non-ideal system is bounded only for low values of the scaling factor, leading to limitations in the implementation phase. The influence of the variation of the scaling factor, on both first and second order statistics of the system state variables, show that the implementable system can still be used as an analogue noise generator.

Further research is needed to perform a similar analysis for the amplifiers implementing the system coefficients. Taking into account the
dynamic behavior of the circuit elements implementing the algebraic calculations in the state equations (e.g. amplifiers, summers, multipliers) is also necessary at the pre-implementation stage.

REFERENCES


INFLUIENȚA NE-IDEALITĂȚILOR DE IMPLEMENTARE ASUPRA DINAMICII NELINIARE A GENERATOARELOR HAOTICE

(Rezumat)

În acest articol, se analizează influența pe care o are modificarea neliniarității sistemului haotic Rossler, asupra comportărilor neliniare a acestuia și asupra proprietăților statistice ale semnalelor generate de sistem. Modificarea neliniarității sistemului Rossler, din produsul a două variabile de stare în produsul tangențelor hiperbolice ale acestora, provine din dorința de a implementa sistemul într-un circuit electronic analogic, în scopul utilizării acestuia ca generator de semnal aleator. Multiplicatorul analogic vizat este o celulă Gilbert realizată cu tranzistoare bipolare. În articol se prezintă ecuația celulei Gilbert și modul în care influențează ecuațiile de stare ale sistemului haotic Rossler. Intrările și ieșirea multiplicatorului analogic sunt scalate cu un coeficient, la variația căruia se studiază comportarea generatorului haotic. Utilizându-se metoda diagramelor de bifurcație, se analizează prin simulare modificarea comportării dinamice neliniare a sistemului Rossler, pentru coeficienți de scalare diferiți. Se evidențiază faptul că, pentru o gamă dinamică suficient de mare a coeficientului de scalare, comportarea generatorului rămâne haotică. Sunt discutate și tipurile de comportări periodice care se obțin la depășirea gamelor dinamice a coeficientului de scalare pentru care generatorul rămâne haotic. De asemenea sunt prezentate simulări privitoare la modificarea proprietăților statistice ale semnalelor generate de sistemul implementat, pentru diverse valori ale coeficientului de scalare. În această serie de simulări, se estimează desititatea de probabilitate prin metoda histogramei, iar densitatea spectrală de putere prin metoda periodogramei. Rezultatele obținute arată că performanțele statistice ale generatorului se păstrează destul de aproape de cele ideale pentru valori suficiente de mici ale factorului de scalare.